Abstract

In this series of talks we introduce and discuss tools of ergodic theory such as recurrence theorems in order to give the proof of Szemeredi's theorem.

In 1927 van der Waerden proved the conjecture of Baudet about arithmetic progressions with the following theorem:

If we colour the set of integers with a finite number of colours then there exists a k-term monochromatic arithmetic progression, for any $k \in \mathbb{N}$.

In 1936 Paul Erdős and Pál Turán conjectured the stronger result that any subset of the natural numbers with positive upper Banach density contains arbitrary long arithmetic progressions. In 1953 Klaus Friedrich Roth proved that a subset of the natural numbers with positive upper Banach density contains 3-term arithmetic progression. In 1969 Endre Szemerédi proved that any set of positive upper Banach density contains 4-term arithmetic progressions and finally in 1975 Szemerédi proved that all such sets contain arbitrarily long arithmetic progressions.

In 1977 Hillel Furstenberg proved Szemerédi's theorem using ergotheoretical tools, and his work gave rise to ergodic Ramsey theory, where one uses tools from ergodic theory to investigate problems in additive combinatorics.

For the proof of the aforementioned theorem first we need to translate the problem of arithmetic progressions to a problem of dynamical systems. This is achieved by using Furstenberg's correspondence principle, an important technique that connects the combinatorial problem with a measure preserving system. Then we give the proof of Szemerédi's theorem in terms of dynamical systems as a consequence of a multiple recurrence theorem.

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